



# Number Theory Online 2021

February 18th, 2021

February 24th, 2021

March 4th, 2021

## Titles and abstracts

### *Organizers*

Andrea Bandini, Università degli Studi di Pisa

Alessandro Gambini, Sapienza Università di Roma

Alessandro Zaccagnini, Università degli Studi di Parma

# PLENARY SPEAKERS

## Recent results on Greenberg transform

*Alessandra Bertapelle*

Institution: *Università degli Studi di Padova - Dipartimento di Matematica "Tullio Levi-Civita" - Italia*

e-mail: [alessandra.bertapelle@unipd.it](mailto:alessandra.bertapelle@unipd.it)

**ABSTRACT:** Greenberg transform (or Greenberg realization) was firstly introduced by S. Lang in his thesis and later studied by his student M.J. Greenberg in [4]. It is an useful tool when studying polynomial equations over an artinian local ring; indeed the Greenberg transform allows to work with associated polynomial equations over the residue field without losing important information. In this talk, after recalling the classical construction and applications, I will present some new results obtained in recent papers [3, 2] (joint with T. Suzuki, E. Previato and A. Saha).

### References

- [1] A. BERTAPELLE, C. D. GONZÁLEZ-AVILÉS, *The Greenberg functor revisited*, European Journal of Math. **4** (2018), 1340–1389.
- [2] A. BERTAPELLE, E. PREVIATO, A. SAHA, *Arithmetic jet spaces*, arXiv:2003.12269 [math.AG] (2020).
- [3] A. BERTAPELLE, T. SUZUKI, *The relatively perfect Greenberg transform and cycle class maps*, arXiv:2009.05084 [math.NT] (2020)
- [4] M. J. GREENBERG, *Schemata over local rings*, Ann. of Math. **73** (1961), 624–648.

## Semi-abelian analogues of Schanuel Conjecture and applications

*Cristiana Bertolin\*, Patrice Philippon, Biswajyoti Saha, Ekata Saha*

Institution: *Dipartimento di Matematica, Università di Torino, Via Carlo Alberto 10, Italy*

e-mail: [cristiana.bertolin@unito.it](mailto:cristiana.bertolin@unito.it)

**ABSTRACT:** In this talk we state Semi-abelian analogues of Schanuel conjecture and then we study their consequences. In particular we will see that a Relative Semi-abelian conjecture implies the algebraic independence of the values of the iterated semi-abelian exponential and the values of an iterated generalized semi-abelian logarithm.

# $p$ -adic deformations and the Birch and Swinnerton-Dyer conjecture

*Massimo Bertolini*

Institution: *Universität Duisburg-Essen, Fakultät für Mathematik, Germany*

e-mail: massimo.bertolini@uni-due.de

**ABSTRACT:** I plan to report on recent results on the Birch and Swinnerton-Dyer conjecture for modular elliptic curves over certain finite extensions of the rational numbers. The central values of the relevant  $L$ -functions are related to rational points on elliptic curves via  $p$ -adic families of cohomology classes arising from geometry.

## Modularity and distribution of quantum knots invariants

*Sandro Bettin*

Institution: *Università di Genova*

e-mail: bettin@dima.unige.it

**ABSTRACT:** We consider Zagier's modularity conjecture for the colored Jones polynomials of hyperbolic knots. We prove this conjecture in some cases and show that, in the case of the  $4_1$  knot, one can also deduce a law of large for the values of the colored Jones polynomial at roots of unity. This is joint work with Sary Drappeau.

## On property (N) and local degrees

*Sara Checcoli\*, Arno Fehm*

Institution: *Institut Fourier, Université Grenoble Alpes, France*

e-mail: sara.checcoli@univ-grenoble-alpes.fr

**ABSTRACT:** A field of algebraic numbers has the Northcott property (N) if it contains only finitely many elements of bounded absolute logarithmic Weil height. While for number fields property (N) follows immediately by Northcott's theorem, to decide property (N) for an infinite extension of the rationals is, in general, a difficult problem. This property was introduced in 2001 by Bombieri and Zannier, who raised the question of whether it holds for fields with uniformly bounded local degrees. They also remarked that, for a (possibly infinite) Galois extension of the rationals whose local degrees are bounded at at least one prime, property (N) is implied by the divergence of a certain sum, but suggested that this phenomenon might occur only for number fields. In 2011 Widmer gave a criterion for an infinite extension of the rationals to have property (N) under some condition on the growth of the discriminants of certain finite subextensions of the field. In this talk we show the existence of infinite Galois extensions of the rationals for which the sum considered by Bombieri and Zannier is divergent and to which Widmer's criterion does not apply. We also show the existence of fields without property (N) and having (non-uniformly) bounded local degrees at all primes. This is joint work with Arno Fehm.

# On the Galois module structure of field extensions: the classical and the Hopf-Galois structures.

*Ilaria Del Corso*

*(with E.Campedell, A.Caranti, F.Ferri, D.Lombardo)*

Institution: *Dipartimento di Matematica Università di Pisa*

e-mail: [ilaria.delcorso@unipi.it](mailto:ilaria.delcorso@unipi.it)

**ABSTRACT:** Let  $L/K$  be a Galois extension with Galois group  $G$ . The Normal Basis Theorem shows that  $L$  is a free  $K[G]$ -module of rank 1. When  $L/K$  is a number field or a local field extension, it is natural to consider the question of determining the structure of the ring of integers  $\mathcal{O}_L$  as  $\mathcal{O}_K[G]$ -module, and this question has long been studied. In this talk I will present an overview of the main classical results and some recent research directions on this subject. We will make a focus on results and important questions concerning both the "classical" Galois structure, and the Hopf-Galois context, whose importance has been growing in the last years.

## Calcolo efficiente della costante di Euler-Kronecker per campi ciclotomici (e problemi collegati)

*Alessandro Languasco*

Institution: *Università di Padova - Dipartimento di Matematica - Italia*

e-mail: [alessandro.languasco@unipd.it](mailto:alessandro.languasco@unipd.it)

**ABSTRACT:** Viene presentato un algoritmo di calcolo efficiente per la costante di Euler-Kronecker per campi ciclotomici che ha portato a permetterne il calcolo per ogni primo dispari  $q \leq 10^7$  (in precedenza erano note per  $q \leq 50\,000$ ) e a trovare nuovi controesempi alla congettura di Ihara dell'ordine da 10 a 50 volte più grandi dei precedenti. Nel contempo si è potuto studiare, per  $q$  nel medesimo intervallo, l'ordine di grandezza di  $\min_{\chi \neq \chi_0} |L'/L(1, \chi)|$  e  $\max_{\chi \neq \chi_0} |L'/L(1, \chi)|$ , dove con  $L(s, \chi)$  si denotano le funzioni  $L$  di Dirichlet e con  $\chi_0$  il carattere principale di Dirichlet. Con idee analoghe si è studiato l'ordine di grandezza di  $|L(1, \chi)|$  e del Kummer ratio per campi ciclotomici. Infine, algoritmi veloci per il calcolo di funzioni speciali aventi proprietà analoghe alla  $\Gamma$  di Euler sono stati sviluppati. Altre conseguenze e applicazioni sono in via di studio.

## References

- [1] Y. LAMZOURI, A. LANGUASCO, *Small values of  $L'/L(1, \chi)$* , Arxiv (2020), <http://arxiv.org/abs/2005.10714>, to appear in *Experimental Mathematics*.
- [2] A. LANGUASCO, *Numerical verification of Littlewood's bounds for  $|L(1, \chi)|$* , Arxiv (2020), <http://arxiv.org/abs/2005.04664>, to appear in *Journal of Number Theory*.

- [3] A. LANGUASCO, *Efficient computation of the Euler-Kronecker constants for prime cyclotomic fields*, *Research in Number Theory* **7** (2) (2021), 1–22.
- [4] A. LANGUASCO, P. MOREE, S. SAAD EDDIN, A. SEDUNOVA, *Computation of the Kummer ratio of the class number for prime cyclotomic fields*, *Arxiv* (2019), <http://arxiv.org/abs/1908.01152>.
- [5] A. LANGUASCO AND L. RIGHI, *A fast algorithm to compute the Ramanujan-Deninger Gamma-function and some number theoretic applications*, *Arxiv*, 2020, <http://arxiv.org/abs/2005.10046>, submitted.

## On the Equivariant Tamagawa Number Conjecture for motives of modular forms

*Matteo Longo*

Institution: *Università di Padova*

e-mail: [mlongo@math.unipd.it](mailto:mlongo@math.unipd.it)

**ABSTRACT:** The Equivariant Tamagawa Number Conjecture was formulated by Bloch and Kato in 1990, and can be seen as a generalisation to motives of the Birch and Swinnerton-Dyer Conjecture for elliptic curves. In the latter case, the validity of the  $p$ -part of the Birch and Swinnerton-Dyer Conjecture for ordinary primes  $p$  is known when the analytic rank of the rational elliptic curve  $E/\mathbb{Q}$  is equal to 1. We prove a similar result for the  $p$ -part of the Bloch-Kato conjecture for motives attached to newforms. For this, we prove a version of Kolyvagin’s Conjecture for modular forms, from which we deduce the  $p$ -part of the Tamagawa Number Conjecture. This is a work in collaboration with Stefano Vigni.

## Classificazione delle funzioni $L$

*Alberto Perelli*

Institution: *Dipartimento di Matematica, Università di Genova*

e-mail: [perelli@dima.unige.it](mailto:perelli@dima.unige.it)

**ABSTRACT:** Parlerò del problema della classificazione delle funzioni  $L$  nell’ambito assiomatico della classe di Selberg, presentando i risultati noti in letteratura relativi ai gradi  $0 \leq d < 2$ ; alla fine presenterò un nuovo risultato relativo al grado  $d = 2$ . Lavoro in collaborazione con J. Kaczorowski.

# $p$ -adic deformations and Stark-Heegner point constructions

*Marco Adamo Seveso*

Institution: *Università degli Studi di Milano - Dipartimento di Matematica - Italia*

e-mail: marco.seveso@gmail.com

## **ABSTRACT:**

### ITALIAN

Sia  $E$  una curva ellittica definita sul campo dei numeri razionali e sia  $p$  un primo di buona riduzione. I punti di Stark-Heegner sono dei punti locali in  $p$  di  $E$  che, congetturalmente, sono globali e giocano un ruolo analogo a quello dei punti di Heegner quando il campo quadratico immaginario è rimpiazzato da un campo quadratico reale. In questo seminario descriverò recenti risultati che esibiscono un legame tra questi punti e deformazioni  $p$ -adiche di classi di coomologia globali e, se ci sarà tempo, generalizzazioni di questo tipo di costruzioni che si applicano anche al caso in cui  $p$  sia un primo di buona riduzione per  $E$ . Si tratta di un lavoro in collaborazione con Massimo Bertolini e Rodolfo Venerucci.

### ENGLISH

Suppose that  $E$  is an elliptic curve defined over the field of rational numbers and that  $p$  is a prime of semistable reduction. The Stark-Heegner points are local points of  $E$  at  $p$  that are conjectured to be global and to play a role analogous to that played by Heegner points when the imaginary field is replaced by a real quadratic field. I plan to report on recent results relating them to  $p$ -adic deformation of global cohomology classes and, time permitting, describe generalizations of this kind of constructions to the case where the prime  $p$  is allowed to be of good reduction for  $E$ . This is a joint work with Massimo Bertolini and Rodolfo Venerucci.

# CONTRIBUTED TALKS

## Ranks of elliptic curves over $\mathbb{Q}(T)$ with small degree in $T$

*Francesco Battistoni*

Institution: *Université Bourgogne Franche-Comté - Laboratoire de Mathématiques de Besançon - France*

e-mail: francesco.battistoni@univ-fcomte.fr

**ABSTRACT:** We consider elliptic curves  $E/\mathbb{Q}(T)$  with coefficients of their Weierstrass equation being polynomials  $\alpha_i(T) \in \mathbb{Q}[T]$  of degree at most 2. The goal of this seminar is to present an explicit expression for the rank of  $E$  depending only on the coefficients  $\alpha_i(T)$ : the computations rely on Nagao's conjecture for rational elliptic surfaces, which is proven for the considered curves, and on the use of prime densities in order to evaluate sums of Legendre symbols. We also give sharp estimates for the ranks of these curves, lists of rational points and examples of curves with maximal rank. This is a joint work with Sandro Bettin (Università di Genova), Chantal David (Concordia University) and Christophe Delaunay (Université de Franche-Comté).

## Cesàro averages of counting functions

*Marco Cantarini*

Institution: *Università Politecnica delle Marche*

e-mail: m.cantarini@univpm.it

**ABSTRACT:** The study of behavior of average of arithmetical functions is classical in number theory. A way to attack these type of problems is to insert a weight with suitable hypotheses into the average, with the aim of making the involved functions "more regular". In this talk we will analyze the Cesàro average, that is, averages of the form

$$\frac{1}{\Gamma(k+1)} \sum_{n \leq N} f(n) \left( \frac{N-n}{N} \right)^k, \quad k \in \mathbb{R}_0^+, N \in \mathbb{N}^+.$$

We will show how this approach has proved, in a series of works developed in recent years up to the present day, to be a versatile tool for the study of these kind of problems for functions  $f(n)$  that count the number of representations of an integer as a sum of elements that belong to some subsets of  $\mathbb{N}$  (for example, powers of prime numbers).

# Multiplicative and linear dependence in finite fields and on elliptic curves modulo primes

*F. Barroero, L. Capuano\*, L. Mérai, A. Ostafe, M. Sha*

Institution: *Politecnico di Torino - Dipartimento di Matematica L. Lagrange - Italian*

e-mail: [laura.capuano@polito.it](mailto:laura.capuano@polito.it)

**ABSTRACT:** In 2008 Maurin proved that given  $n$  multiplicatively independent rational functions  $\varphi_1(x), \dots, \varphi_n(x) \in \mathbb{Q}(x)$ , there are at most finitely many  $\alpha \in \overline{\mathbb{Q}}$  such that  $\varphi_1(\alpha), \dots, \varphi_n(\alpha)$  satisfy two independent multiplicative relations. This statement is an instance of more general conjectures of *unlikely intersections* over tori made by Bombieri, Masser and Zannier and independently by Zilber. We consider a positive characteristic variant of this problem, proving that, for sufficiently large primes, the cardinality of the set of  $\alpha \in \overline{\mathbb{F}}_p$  such that  $\varphi_1(\alpha), \dots, \varphi_n(\alpha)$  satisfy two independent multiplicative relations with exponents bounded by a certain constant  $K$  is bounded independently of  $K$  and  $p$ . We prove analogous results for products of elliptic curves and for split semiabelian varieties  $E^n \times \mathbb{G}_m^k$ .

# Sul volume nodale di onde aritmetiche aleatorie

*Giacomo Cherubini*

Institution: *Charles University, Faculty of Mathematics and Physics, Department of Algebra, Sokolovská 83, 18600, Prague, Czech Republic*

e-mail: [cherubini@karlin.mff.cuni.cz](mailto:cherubini@karlin.mff.cuni.cz)

**ABSTRACT:** Sul toro  $d$ -dimensionale  $\mathbb{R}^d/\mathbb{Z}^d$  studiamo combinazioni lineari di onde di base (seni e coseni a frequenze intere) con coefficienti che sono variabili aleatorie Gaussiane (*onde aritmetiche aleatorie*). Gli insiemi di livello zero di tali onde, e in particolare il loro volume, sono legati allo studio delle correlazioni lineari di punti interi sulla sfera di raggio  $\sqrt{n}$  in  $\mathbb{R}^d$ . Il problema è stato risolto in dimensione  $d = 2, 3$  utilizzando risultati di teoria dei numeri e di teoria dei grafi. Al crescere della dimensione queste tecniche lasciano il posto all'analisi. Discutiamo in questo seminario ciò che è noto in dimensione  $d \geq 4$  e le applicazioni al volume degli insiemi di livello zero delle onde aritmetiche aleatorie. Lavoro in collaborazione con N. Laaksonen.

# On reduction steps for Leopoldt's conjecture

*Fabio Ferri*

Institution: *University of Exeter, Department of Mathematics, United Kingdom*

e-mail: ff263@exeter.ac.uk

**ABSTRACT:** Let  $p$  be a rational prime and let  $L/K$  be a Galois extension of number fields with Galois group  $G$ . Under some hypotheses, we show that Leopoldt's conjecture at  $p$  for certain proper intermediate fields of  $L/K$  implies Leopoldt's conjecture at  $p$  for  $L$ ; a crucial tool will be the theory of norm relations in  $\mathbb{Q}[G]$ . We also consider relations between the Leopoldt defects at  $p$  for intermediate extensions of  $L/K$ , noting that Leopoldt kernels form a cohomological Mackey functor. We finally show that our results combined with some techniques introduced by Buchmann and Sands allow us to find infinite families of nonabelian totally real Galois extension of  $\mathbb{Q}$  satisfying Leopoldt's conjecture for certain primes. This is joint work with Henri Johnston.

# Some weighted analyses of the Riemann zeta function

*Alessandro Fazzari*

Institution: *Università di Genova - Dipartimento di Matematica - Italia*

e-mail: fazzari@dima.unige.it

**ABSTRACT:** We investigate some weighted aspects regarding the value distribution of the Riemann zeta function on the critical line and the linear statistics of its nontrivial zeros, with the aim of studying the large values of zeta. The prototypical example is a weighted analogue of Selberg's central limit theorem, showing that  $\log |\zeta(1/2 + it)|$  has an approximately normal distribution also with respect to the tilted measure  $|\zeta(1/2 + it)|^{2k} dt$ , for  $k \in \mathbb{N}$ , as  $t \in [T, 2T]$  and  $T \rightarrow \infty$ , assuming the Riemann Hypothesis and the asymptotic formula for the twisted and shifted integral moments of zeta.

# Generalized Heegner cycles over Shimura curves and $p$ -adic $L$ -functions

*Paola Magrone*

Institution: *Università degli Studi di Genova - Dipartimento di Matematica - Italia*

e-mail: magrone@dima.unige.it

**ABSTRACT:** In [3], Castella and Hsieh proved significant results about Selmer groups associated with Galois representations attached to newforms twisted by Hecke characters of an imaginary quadratic field. These results lead also to prove other instances of the rank 0 case of the Bloch–Kato Conjecture for  $L$ -functions of modular forms. The key point of the work of Castella and Hsieh is a remarkable link between certain arithmetic objects called *generalized Heegner cycles* that were introduced by Bertolini, Darmon and Prasanna in [1] and suitably defined  $p$ -adic  $L$ -functions, which are instead objects of  $p$ -adic analytic nature, interpolating special values of complex  $L$ -series. All these results are obtained under the so-called *Heegner hypothesis* that the imaginary quadratic field must satisfy with respect to the level of the modular form. But what happens if we want to weaken the Heegner hypothesis, considering more quadratic fields? In this talk, we see that several of the results of Castella–Hsieh can be extended to a quaternionic setting, that is the setting that arises when one works under a “relaxed” Heegner hypothesis. This can be done working with *generalized Heegner cycles* over Shimura curves (instead of modular curves) introduced by Brooks in [2]. For all the details, see [4].

## References

- [1] M. BERTOLINI, H. DARMON, K. PRASANNA, *Generalized Heegner cycles and  $p$ -adic Rankin  $L$ -series*, Duke Math. J. **162** (2013), no. 6, 1033–1148.
- [2] E. H. BROOKS, *Shimura curves and special values of  $p$ -adic  $L$ -functions*, Int. Math. Res. Not. IMRN (2015), no. 12, 4177–4241.
- [3] F. CASTELLA, M. HSIEH, *Heegner cycles and  $p$ -adic  $L$ -functions*, Math. Ann. **370**, Springer (2018), Issue 1–2, 567–628.
- [4] P. MAGRONE, *Generalized Heegner cycles and  $p$ -adic  $L$ -functions in a quaternionic setting*, preprint, arXiv:2008.13500.

# On maximal product sets of random sets

*Daniele Mastrostefano*

Institution: *University of Warwick, Mathematics Institute, Zeeman Building, Coventry, CV4 7AL, UK*

e-mail: [Daniele.Mastrostefano@warwick.ac.uk](mailto:Daniele.Mastrostefano@warwick.ac.uk)

**ABSTRACT:** For every positive integer  $N$  and every  $\alpha \in [0, 1)$ , let  $B(N, \alpha)$  denote the probabilistic model in which a random set  $A \subset \{1, \dots, N\}$  is constructed by choosing independently every element of  $\{1, \dots, N\}$  with probability  $\alpha$ . We prove that, as  $N \rightarrow +\infty$ , for every  $A$  in  $B(N, \alpha)$  we have  $|AA| \sim |A|^2/2$  with probability  $1 - o(1)$ , if and only if

$$\frac{\log(\alpha^2(\log N)^{\log 4-1})}{\sqrt{\log \log N}} \rightarrow -\infty.$$

This improves on a theorem of Cilleruelo, Ramana and Ramaré, who proved the above asymptotic between  $|AA|$  and  $|A|^2/2$  when  $\alpha = o(1/\sqrt{\log N})$ , and supplies a complete characterization of maximal product sets of random sets.

# Random behavior of deterministic sequences: from global scale to local scale

*Marc Munsch*

Institution: *TU Graz, Austria*

e-mail: [munsch@math.tugraz.at](mailto:munsch@math.tugraz.at)

**ABSTRACT:** The uniform distribution modulo 1 of a sequence  $\{x_n\}_{n \geq 1}$  measures the pseudo-random behavior at a global scale and can be characterized using exponential sums. At a more localized scale, we can study the pair correlation for sequences in the unit interval. Pseudo-random behavior with respect to this statistic is called Poissonian behavior. The metric theory of pair correlations of sequences of the form  $(a_n \alpha)_{n \geq 1}$  has been pioneered by Rudnick, Sarnak and Zaharescu. Recently, a general framework was developed which gives a criterion for Poissonian pair correlation of such sequences for almost  $\alpha \in (0, 1)$ , in terms of the additive energy of the integer sequence  $\{a_n\}_{n \geq 1}$ . In the present talk we will discuss a similar framework in the more delicate case where  $\{a_n\}_{n \geq 1}$  is a sequence of reals. We give several concrete applications of our method and present some open problems. This is a joint work with Christoph Aistleitner and Daniel El-Baz.

## On the $\mathcal{L}$ -invariant for Hilbert modular forms

*Maria Rosaria Pati*

Institution: *Università degli Studi di Genova*

e-mail: [mariarosariapati@gmail.com](mailto:mariarosariapati@gmail.com)

**ABSTRACT:** Let  $\mathbf{f} \in S_{k,w}(\Gamma_0(\mathfrak{n}))$  be a primitive Hilbert modular cuspform and  $p$  a rational prime. The  $p$ -adic analytic  $L$ -function  $L_p(\mathbf{f}, s)$  attached to  $\mathbf{f}$  contains extra interpolation factors that can possibly vanish at the central critical point of  $\mathbf{f}$ . In this event, the discrepancy between  $L_p(\mathbf{f}, s)$  at the central point and the special value of the complex  $L$ -function is accounted by the so-called  $\mathcal{L}$ -invariant of  $\mathbf{f}$ . It has several a priori different definitions, all conjecturally equivalent.

We give a definition of the  $\mathcal{L}$ -invariant of  $\mathbf{f}$  by lifting  $\mathbf{f}$  to a modular form on a quaternionic Shimura variety and by studying the cohomology of such variety with coefficients in a suitable space of polynomials. We prove that our  $\mathcal{L}$ -invariant is equal to the Fontaine-Mazur  $\mathcal{L}$ -invariant.

# Universal norms of $p$ -adic Galois representations and the Fargues-Fontaine curve

*Gautier Ponsinet*

Institution: *Università degli Studi di Genova - Dipartimento di Matematica - Italia*

e-mail: [gponsinet@gmail.com](mailto:gponsinet@gmail.com)

**ABSTRACT:** In 1996, Coates and Greenberg computed the module of universal norms associated with an abelian variety in a perfectoid field extension. A precise description of this module is essential in Iwasawa theory, notably to study Selmer groups over infinite algebraic fields extensions. Coates and Greenberg then asked if their result could be generalised to other motives. In this talk, I will present a new approach to this question relying on the classification of vector bundles over the Fargues-Fontaine curve, which enables to answer it affirmatively in new cases.

## References

- [1] L. BERGER, *Représentations de de Rham et normes universelles*, Bull. Soc. Math. France **133** (2005), 601–618.
- [2] J. COATES AND R. GREENBERG, *Kummer theory for abelian varieties over local fields*, Invent. Math. **124** (1996), 129–174.
- [3] L. FARGUES AND J.-M. FONTAINE, *Courbes et fibrés vectoriels en théorie de Hodge  $p$ -adique*, Astérisque, **406** (2018), with a preface by P. Colmez.
- [4] G. PONSINET, *Universal norms and the Fargues-Fontaine curve*, Preprint (2020), available on <https://arxiv.org/abs/2010.02292>.

## Least Common Multiples of Integer Sequences: Old and New Results

*Carlo Sanna*

Institution: *Politecnico di Torino - Department of Mathematical Sciences - Italy*

e-mail: [carlo.sanna.dev@gmail.com](mailto:carlo.sanna.dev@gmail.com)

**ABSTRACT:** A consequence of the Prime Number Theorem is the asymptotic formula

$$\log \operatorname{lcm}(1, 2, \dots, n) \sim n, \quad \text{as } n \rightarrow +\infty,$$

where  $\log$  is the natural logarithm and  $\operatorname{lcm}$  denotes the least common multiple. Motivated by that, many authors have considered asymptotic formulas regarding  $\operatorname{lcm}(a_1, a_2, \dots, a_n)$ , where  $(a_n)_{n \geq 1}$  is an integer sequence of a particular kind; for example, a polynomial sequence or a linear recurrence. This talk will illustrate several results on this topic, including variations on random sequences and  $q$ -analogs.

# Group cohomology and elliptic curves

*Sebastiano Tronto*

Institution: *Department of Mathematics - University of Luxembourg - Luxembourg*

e-mail: [sebastiano.tronto@uni.lu](mailto:sebastiano.tronto@uni.lu)

**ABSTRACT:** In this talk I will present a recent – or rather, still in progress – joint work with Davide Lombardo. Rather than presenting the main result of the paper, I will try to make a self-contained and detailed presentation of one particular result about a certain cohomology group associated with elliptic curves.

More precisely, let  $E$  be an elliptic curve defined over  $\mathbb{Q}$  and let  $\mathbb{Q}_\infty$  be the number field generated by the coordinates of all torsion points of  $E$ . Denoting by  $G$  the Galois group of  $\mathbb{Q}_\infty$  over  $\mathbb{Q}$  and by  $E_t$  the group of all torsion points of  $E$  (with coordinates in an algebraic closure of  $\mathbb{Q}$ ), we proved that the exponent of  $H^1(G, E_t)$  divides a universal constant  $C$ , and we were able to provide an explicit value for  $C$ .

## *GCD* results on semiabelian varieties and a conjecture of Silverman

*Fabrizio Barroero, Laura Capuano, Amos Turchet (\*)*

Institution: *Università degli Studi Roma 3 - Dipartimento di Matematica e Fisica - Italy*

e-mail: [amos.turchet@uniroma3.it](mailto:amos.turchet@uniroma3.it)

**ABSTRACT:** A divisibility sequence is a sequence of integers  $d_n$  such that whenever  $m$  divides  $n$  one has that  $d_m$  divides  $d_n$ . Results of Bugeaud, Corvaja, Zannier, among others, have shown that pairs of divisibility sequences corresponding to subgroups of the multiplicative group have only limited common factors. Silverman conjectured that a similar behaviour should appear in (a large class of) all algebraic groups. We show, extending work of Ghioca-Hsia-Tucker and Silverman on elliptic curves, how to prove the analogue of Silverman's conjecture over function fields for abelian and split semi-abelian varieties and some generalizations.

# Atkin-Lehner theory for Drinfeld Modular forms

*Maria Valentino*

Institution: *Università della Calabria - Dipartimento di Matematica ed Informatica  
- Italia*

e-mail: maria.valentino@unical.it

**ABSTRACT:** Let  $S_k(N)$  be the space of cusp forms of level  $\Gamma_0(N)$  with  $N \in \mathbb{Z}$ . Atkin-Lehner theory deals with the notion of oldforms, namely those coming from a lower level  $M|N$ , and newforms, i.e. the orthogonal complement of the space of oldforms with respect to the Petersson inner product. Moreover, it is also concerned with the construction of a basis for  $S_k(N)$  made up by eigenfunctions for the Hecke operators  $\mathbf{T}_n$  with  $n$  prime to  $N$ . In this talk we shall present some recent advances on the theory for Drinfeld modular forms, which are the analogue over  $\mathbb{F}_q[T]$  of classical modular forms.

# Participants

Anshid Aboobacker, BITS Pilani Hyderabad Campus  
Jessica Alessandrì, Università degli Studi dell'Aquila  
Fares AlHarbi, King Saud University  
Sabrina Amico, Studente  
Ivan Andreoni, UniMi  
Dario Antolini, Università Tor Vergata  
Paresh Arora, National Institute of Science Education and Research  
Óscary Ávila-Hernández, Autonomous University (UNAB) - University of The Andes  
Francesco Baldassarri, University of Padova  
Andrea Bandini, Università di Pisa  
Marco Baracchini, Student at Università di Genova  
Jiban Barman, Jadavpur University  
Fabrizio Barroero, Università degli Studi Roma Tre  
Francesco Battistoni, Université Bourgogne Franche-Comté  
Abdelmejid Bayad, Université Paris-Saclay  
Danilo Bazzanella, Politecnico di Torino  
Andreas Bender, Università di Pavia  
Ranjan Bera, IIT, Hyderabad  
Elena Berardini, École Polytechnique  
Yamid Bermudez, Universidad del Valle, Cali Colombia  
Alessandra Bertapelle, Università degli Studi di Padova  
Cristiana Bertolin, Università degli Studi di Torino  
Massimo Bertolini, Universität Duisburg-Essen  
Sandro Bettin, Università di Genova  
Shubhrajit Bhattacharya, Chennai Mathematical Institute  
Damanvir Binner, Simon Fraser University  
Werner Bley, LMU München  
Kajtaz Bllaca, University of Prishtina  
Matteo Bordignon, University of New South Wales Canberra  
Armand Brumer, Fordham University  
Francesco Campagna, University of Copenhagen  
Elena Campedel, Università degli studi di Milano-Bicocca  
Stefano Canino, Politecnico di Torino  
Marco Cantarini, Università Politecnica delle Marche  
Victoria Cantoral, KU Leuven  
Laura Capuano, Politecnico di Torino  
Andrea Caranti, Department of Mathematics, University of Trento, Italy  
Annarosa Castellano, Politecnico di Torino  
Mattia Cavicchi, IRMA Strasbourg  
Dimitrios Chatzakos, University of Bordeaux  
Sara Checcoli, Institut Fourier, Université Grenoble-Alpes  
Giacomo Cherubini, Charles University  
Masataka Chida, Tokyo Denki University  
Shiva Chidambaram, University of Chicago  
Mihai Cipu, Institute of Mathematics Romanian Academy Bucharest  
Alessandro Cobbe, Universität der Bundeswehr München  
Nirvana Coppola, University of Bristol

Naoto Dainobu, Keio University  
Tarun Dalal, IIT Hyderabad  
Luca Dall'Ava, Universität Duisburg-Essen  
Ilaria Del Corso, Università di Pisa  
Vincenzo Di Bartolo, Università di Genova  
Gabriel Dill, University of Oxford  
Asset Durmagambetov, L.N. Gumilyov Eurasian National University  
Shashikant Dwivedi, Avadh degree college  
Silvia Fabiani, Università di Pisa  
Tommaso Faustini, Università di Pisa  
Alessandro Fazzari, Università di Genova  
Andrea Feraguti, Scuola Normale Superiore  
Fabio Ferri, University of Exeter  
Jonas Franzel, Universität Duisburg-Essen  
Reynold Fregoli, University of York  
Linda Frey, University of Copenhagen  
Swati Gaba, University of Illinois at Chicago  
Franco Galvagno, PoliTO  
Alessandro Gambini, Sapienza Università di Roma  
Ke Gong, Henan University  
Pip Goodman, University of Bristol  
Heidi Goodson, Brooklyn College, CUNY  
Georges Grekos, Mathématiques Université Saint-Etienne, France  
Kalpok Guha, Presidency University  
Shota Inoue, Nagoya University  
Tomasz Jędrzejak, University of Szczecin  
Rania Kammoun, Univeristy Hafr Al Batin Ksa and University of Sfax Tunisia  
Debasish Karmakar, Harish-Chandra Research Institute  
Atsushi Katsuda, Kyushu university  
Ananyo Kazi, University of Milan  
Utkal Keshari Dutta, Department of Mathematics, Sambalpur University, India  
Suraj Khurana, Harish-Chandra Research Institute  
Masanari Kida, Tokyo University of Science  
Chan-Ho Kim, KIAS  
Seoyoung Kim, Queen's University  
Naoki Kimura, Waseda University  
Shinichi Kobayashi, Kyushu University  
Narasimha Kumar, Indian Institute of Technology Hyderabad  
Veerendra Kumar, Central University of South Bihar, Gaya, India  
Sunil Kumar Pasupulati, IISER TVM  
Prasanta Kumar Ray, Sambalpur University, India  
Alessandro Languasco, Università di Padova  
Antonio Lei, Laval University  
Paolo Leonetti, Università Bocconi  
Rusen Li, Shandong University  
Yongxiao Lin, EPFL  
Davide Lombardo, Università di Pisa  
Ignazio Longhi, Indian Institute of Science  
Matteo Longo, Università di Padova

Cynthia Ma, Independent researcher  
Paola Magrone, Università degli Studi di Genova  
Saikat Maity, University of Calcutta  
Priyanka Majumder, TU Darmstadt  
Jamal Maleki, myself  
Luca Marannino, Universität Duisburg-Essen  
Luca Mastella, Università di Padova  
Daniele Mastrostefano, University of Warwick  
Yuki Matsuoka, NTT DATA Mathematical Systems Inc.  
Kamel Mazhouda, University of Monastir, Tunisia  
Nicola Mazzari, Università degli Studi di Padova  
Mentzelos Melistas, University of Georgia  
Pietro Mercuri, Università di Udine  
Aradhana Mishra, A.P.S. University Rewa  
Ali Mkhida, Groningen University  
Giuseppe Molteni, Dipartimento di Matematica Università di Milano  
Andrea Mori, Università di Torino  
Marzio Mula, Università degli Studi di Trento  
Marc Munsch, TU Graz  
Simone Muselli, University of Bristol  
Subramani Muthukrishnan, Indian Institute of Information Technology D&M  
Francesco Naccarato, Scuola Normale Superiore  
Evangelos Nastas, SUNY  
Florent Nguema Ndong, Université des Sciences et Techniques de Masuku  
Filippo A. E. Nuccio, Univ de Lyon - UJM - Saint-Étienne (Francia)  
Robert Osburn, University College Dublin  
Alina Ostafe, UNSW Sydney  
Kazuto Ota, Osaka University  
Tim Page, The University of Manchester  
Laura Paladino, Università della Calabria  
Deepika Parmar, IIT-Gandhinagar  
Maria Rosaria Pati, Università degli Studi di Genova  
Federico Pellarin, Université de Saint-Etienne, Faculté de Sciences  
Riccardo Pengo, École normale supérieure de Lyon  
Alberto Perelli, Università di Genova  
Antonella Perucca, University of Luxembourg  
Gautier Ponsinet, Università degli Studi di Genova & INdAM  
Yang Pu Justin, UBC  
Anwesh Ray, University of British Columbia  
Francesca Rizzo, University of Pisa  
Sajad Salami, Universidade Estadual do Rio de Janeiro  
Carlo Sanna, Politecnico di Torino  
Abhishek Sarma, Tezpur University (Alumni)  
Andrea Sartori, Tel Aviv University  
Marco Adamo Seveso, Università degli Studi di Milano  
Francesco Sica, Nazarbayev University  
Naveen Somasunderam, State University of New York, Plattsburgh  
Luca Speciale, Università di Pisa  
Lorenzo Stefanello, Università di Pisa

Sam Streeter, University of Bath  
Chacha (Ade Irma Suriajaya), Kyushu University  
Masatoshi Suzuki, Tokyo Institute of Technology  
Valerio Talamanca, Roma Tre  
Lea Terracini, Università di Torino  
Emiliano Torti, University of Luxembourg  
Daniele Troletti, Università degli Studi di Padova  
Emanuele Tron, Institut de Mathématiques de Bordeaux  
Sebastiano Tronto, University of Luxembourg  
Kim Tuan Do, Princeton University  
Amos Turchet, Università degli studi Roma Tre  
Murat Uyar, Sabanci University  
Maria Valentino, Università della Calabria  
Francesco Veneziano, Università di Genova  
Matteo Verzobio, Università di Pisa  
Ilaria Viglino  
Stefano Vigni, Università degli Studi di Genova  
Isao Wakabayashi, Seikei University  
Nihal Yurdakul, Sabanci University  
Alessandro Zaccagnini, Università di Parma  
Sadiah Zahoor, Univeristy of Sheffield  
Francesco Zerman, Università di Genova  
Di Zhang, University of Sheffield  
洋太 前田, Kyoto University  
良 定松, 株式会社アウトソーシングテクノロジー