4th Number Theory Ph.D Students Day

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Elena Berardini (École Polytechnique, Paris)

Algebraic geometry codes from surfaces over finite fields

Algebraic geometry codes were introduced by Goppa in 1981on curves defined over finite fields and were extensively studied since then. Even though Goppa construction holds on varieties of dimension higher than one, the literature is less abundant in this context. However, some work has been undertaken on codes from surfaces. The goal of this talk is to provide a theoretical study of algebraic geometry codes constructed from surfaces defined over finite fields. This required using tools from intersection theory and bounds on the number of rational points on curves over surfaces. First, we give lower bounds for the minimum distance of algebraic geometry codes in two wide families of surfaces: those with strictly-nef or anti-nef canonical divisor, and those which do not contain absolutely irreducible curves of small genus. Secondly, we specialize and improve these bounds for particular families of surfaces, for instance in the case of abelian surfaces and minimal fibrations, using the geometry of these surfaces.
The results appear in two joint works with Y. Aubry, F. Herbaut and M. Perret.

Matteo Bordignon (University of New South Wales Canberra)

Some new results on the explicit Pólya-Vinogradov inequality

Taken χ a non-principal Dirichlet character modulus q, the Pólya–Vinogradov inequality states that

$$\sum_{n=1}^{N} \chi(n) \ll \sqrt{q} \log q,$$

for any natural number N. In the talk we will present three recent results regarding the explicit version of the Pólya–Vinogradov inequality: the best explicit version for Pólya–Vinogradov for large primitive characters, one for square-free characters (joint work with Bryce Kerr) and at last how these are explicitly related to Burgess's bound (joint work with Forrest Francis).

References

[1] M. Bordignon and B. Kerr, *An explicit Pólya–Vinogradov inequality via Partial Gaussian sums*, Transactions of the American Mathematical Society, 373:6503–6527, 2020

[2] M. Bordignon, *Partial Gaussian sums and the Pólya–Vinogradov inequality for primitive characters*, submitted, https://arxiv.org/abs/2001.05114, 2019

[3] M. Bordignon and F. Francis, *Explicit Improvements to the Burgess Bound Via Pólya–Vinogradov*, submitted, https://arxiv.org/abs/2007.08119, 2020

Elena Campedel (Università degli Studi di Milano-Bicocca)

Hopf Galois theory: an introduction and classification of the Hopf Galois structures for a Galois extension of degree p^2q
Hopf Galois theory for finite and separable field extensions can be viewed as a generalization of Galois theory, where the Hopf algebra replaces the Galois group. Contrary to the classical case, an extension L/K admits several Hopf Galois structures, and this can happen also when L/K is Galois. This theory recovers many extensions that are not Galois but it remains interesting also in the case of Galois extensions: Hopf Galois structures on Galois extensions are linked both to the Galois module theory and to the skew braces. In this talk I will introduce Hopf Galois theory and I will present some results about the classification of the Hopf Galois structures for a Galois extension of degree p^2q.

Quentin Gazda (Institut Camille Jordan, Lyon and Saint-Étienne)

First Beilinson's conjecture in function fields arithmetic
In the mid 80's, Beilinson formulated deep conjectures relating special values of L-functions to pieces of K-theory, superseding at once the BSD conjecture and Deligne's conjecture. Beilinson's conjectures are fully expressed in the framework of mixed motives, which remains hypothetical. This talk will be devoted to portray the analogous picture in the function fields setting, using so-called Goss L-values instead of classical L-values, and mixed (uniformizable) Anderson A-motives instead of Grothendieck's mixed motives. After a recall of the classical conjectures, we shall discuss and define the analogue of motivic cohomology and regulators for function fields, and express the counter-part of Beilinson's conjectures.

Lorenzo Pagani (Università La Sapienza, Roma)

Greenberg's conjecture for real quadratic fields
Let A_n be the 2-part of the ideal class group of the n-th layer of the cyclotomic $\mathbb{Z}_2$-extension of a real quadratic number field F. The cardinality of A_n is related to the index of cyclotomic units in the full group of units. We present a method to study the latter index. As an application we show that the sequence of the A_n's stabilizes for the real fields F with conductor less than 10000. Equivalently Greenberg's conjecture holds for those fields.

Ilaria Viglino (ETH Zurich)

Splitting properties of certain families of S_n-polynomials and application to class group torsion
It is known that with high probability a random degree n monic polynomial with integer coefficients and height less or equal than N, is irreducible with splitting field over $\mathbb{Q}$ with Galois

group S_n. We are interested in an explicit version of the Chebotarev density theorem on average for our family (and also for subfamilies) of S_n-polynomials, without assuming GRH for the Dedekind zeta-function or the Artin conjecture, because of the average properties of the family. By the work of Ellenberg and Venkatesh, the existence of many primes that split completely in a finite Galois extension, contributes significantly to the quotient of the class group by its ℓ-torsion, yielding interesting upper bounds which improve the trivial bound by the order of the entire class group. It represents an evidence towards the so-called "ϵ-conjecture", suggested by Duke for CM fields and in a stronger form by Brumer and Silverman.